A simple explanation of the National Income Accounts

Wendy A. Compione  
Northern Arizona University wendy.campione@nau.edu
Max E. Jerrell  
Northern Arizona University max.jerrell@nau.edu
James N. Morgan  
Northern Arizona University james.morgan@nau.edu

Undergraduate students often have difficulty understanding the National Income Accounts. We believe that the chief problem is that they cannot understand why factor income should be equal to GDP and why the sum of value added should also equal to GDP. They often feel that this is an interesting concept but of no practical significance. We think these relationships can be easily demonstrated and that the following will give students a better understanding of the NIA’s.

1 Factor income = GDP

In this section we show why factor income should be equal to GDP. Figure 1 is an input–output diagram for a three good, two factor economy. A subscript is used to designate a particular good. $X_1$, $X_2$ and $X_3$ indicate the quantities of the three goods sold during the accounting period. For simplicity assume that the prices of these goods, $P_1$, $P_2$ and $P_3$ respectively, remain unchanged during the accounting period. $X_{13}$ indicates the quantity of good $X_3$ used to produce $X_1$ and $X_{23}$ is the quantity of good $X_3$ used to produce $X_2$. Note that $X_1$ and $X_2$ are final goods and $X_3$ is an intermediate good. For simplicity we assume that the economy is closed and that all output produced is sold during the accounting period. Then GDP is defined to be the value of all final goods and services produced by an economy during an accounting period, so

$$GDP = P_1X_1 + P_2X_2.$$
The factors of production are labor and entrepreneurship. Labor receives a uniform wage rate, $w$, and supplies quantities $L_1$, $L_2$, and $L_3$ of labor to the production of $X_1$, $X_2$, and $X_3$ respectively. Entrepreneurs earn profits $\pi_1$, $\pi_2$, and $\pi_3$ respectively. For this economy then

\begin{align*}
\text{wage income} & = wL_1 + wL_2 + wL_3 \\
\text{entrepreneurs income} & = \pi_1 + \pi_2 + \pi_3 \\
\text{factor income} & = wL_1 + wL_2 + wL_3 + \pi_1 + \pi_2 + \pi_3
\end{align*}

The accounting profits for an industry is given by

$$\pi = \text{total revenue} - \text{total cost}$$

and for the three industries

\begin{align*}
\pi_1 & = P_1X_1 - (wL_1 + P_3X_{13}) \\
\pi_2 & = P_2X_2 - (wL_2 + P_3X_{23}) \\
\pi_3 & = P_3X_3 - wL_3
\end{align*}

(2)

Substituting the expressions for profits into the factor income equation gives

\begin{align*}
\text{factor income} & = wL_1 + wL_2 + wL_3 + \pi_1 + \pi_2 + \pi_3 \\
& = wL_1 + wL_2 + wL_3 \\
& \quad + P_1X_1 - wL_1 - P_3X_{13} \\
& \quad + P_2X_2 - wL_2 - P_3X_{23} \\
& \quad + P_3X_3 - wL_3
\end{align*}

or

$$\text{factor income} = P_1X_1 + P_2X_2 + P_3X_3 - P_3(X_{13} + X_{23}).$$

But $X_{13} + X_{23} = X_3$, so

$$2$$
Figure 1: An input–output diagram

\[
\begin{align*}
\text{factor income} & = P_1 X_1 + P_2 X_2 + P_3 X_3 - P_3 (X_{13} + X_{23}) \\
& = P_1 X_1 + P_2 X_2 + P_3 X_3 - P_3 X_3 \\
& = P_1 X_1 + P_2 X_2 \\
& = GDP.
\end{align*}
\]

2 The sum of value added = GDP

In this section we show that the sum of value added also equals GDP. Defining value added to be total revenue less cost of materials for the three goods gives

\[
\begin{align*}
VA_1 & = P_1 X_1 - P_3 X_{13} \\
VA_2 & = P_2 X_2 - P_3 X_{23} \\
VA_3 & = P_3 X_3.
\end{align*}
\]

The sum of value added is
\[ VA_1 + VA_2 + VA_3 = P_1X_1 - P_3X_{13} + P_2X_2 - P_3X_{23} + P_3X_3 \]
\[ = P_1X_1 + P_2X_2 - P_3(X_{12} + X_{23}) + P_3X_3 \]
\[ = P_1X_1 + P_2X_2 = GDP. \]

Note that intermediate goods fall out of the calculation as they did in Section 1. As a practical matter it is very easy to calculate value added for all firms. Total revenue and cost of materials are items reported to the government for tax purposes. It is not necessary to consider what particular goods a firm produces or the prices charged for the goods. Also it is not necessary to determine if a good is an intermediate good or a final good. Finally, because these values are computed on a yearly basis the problem of price changes for goods during a year are automatically accounted for by using value added.