Profit maximization for a monopolist

- \texttt{reset(); assume(c>0); assume(P<0):}

- \texttt{assume(Qo>0);}
  \( (0, \infty) \)

The monopolist has some pricing power. The monopolist can determine market price by increasing or decreasing the level of output. The resulting price is determined by the demand curve

- \( P := Q \rightarrow Qo - Q \)

Total revenue, as always, is given by the product of price and output. Note however that price is determined by output

- \( TR := Q \rightarrow P(Q) \cdot Q \)

Total cost is given by

- \( TC := Q \rightarrow c \cdot Q^2 \)

where \( c \) is the per unit cost of output. As with the competitive firm, profits are maximized by locating that level of output that make marginal revenue equal to marginal cost.

- \( MR := D(TR) ; \)
  \( Q \rightarrow Qo - 2 \cdot Q \)

- \( MC := D(TC) ; \)
  \( Q \rightarrow 2 \cdot Q \cdot c \)

- \texttt{ans:=solve(MR(Q)=MC(Q),Q) ;}
  \( \left\{ \frac{Qo}{2 \cdot c + 2} \right\} \)

The symbolic solution shows that the optimal level of output increases if \( Qo \) increases (the demand curve shifts right). The optimal level of output will decrease if per unit cost
c increases.

A numerical solution can be obtained if the parameters $Q_0$ and $c$ are given values.

- $Q_0:=1000; c:=10 :$

- $\text{optimal}_Q:=\text{float}(\text{op}(\text{ans})) ;$
  
  $45.45454545$

Next check to see if this value of output is one that makes marginal revenue equal to marginal cost

- $\text{MC}(\text{optimal}_Q) ; \text{MR}(\text{optimal}_Q) ;$

  $909.0909091$

  $909.0909091$

  $909.0909091$

  $909.0909091$
Next a graphical solution for the profit maximizing level of output can be sought.

- \( f_1 := \text{plot::Function2d}(\text{MC}(x), \ x=10..100) \) :
- \( f_2 := \text{plot::Function2d}(\text{MR}(x), \ x=10..100) \) :
- \( f_3 := \text{plot::Line2d}([\text{optimal}_Q,0],[\text{optimal}_Q,\text{MR}(<text{optimal}_Q>),\text{Color}=\text{RGB::Green}) : \)
- \( \text{plot}(f_1,f_2,f_3, \text{AxesTitles}=["Q", "MR,MC"]); \)

Figure 1. Graphically locating the profit maximizing level of output by plotting MR and MC
t is also possible to locate the profit maximizing level of output graphically by plotting the total cost and total revenue functions directly.

- \( f_1 := \text{plot::Function2d}(TR(x), x=10..100, \text{Color=RGB::Red}) \):
- \( f_2 := \text{plot::Function2d}(TC(x), x=10..100, \text{Color=RGB::Green}) \):
- \( f_3 := \text{plot::Line2d}([\text{optimal}_Q,TR(\text{optimal}_Q)],[\text{optimal}_Q,TC(\text{optimal}_Q)], \text{Color=RGB::Blue}) \):
- \( \text{plot}(f_1,f_2,f_3, \text{AxesTitles=}[\text{"Q","TR,TC"]}) ; \)

Figure 2. Graphically locating the profit maximizing level of output by plotting TR and TC
Finally the profit maximizing level of output can be locate graphically by plotting the profit function directly.

• \( \text{Profit} := TR - TC \);
  \[ (Q \rightarrow P(Q) \cdot Q) - (Q \rightarrow c \cdot Q^2) \]

• \( f1 := \text{plot::Function2d} (\text{Profit}(x), x=10..100) \);
• \( f2 := \text{plot::Line2d} ([\text{optimal}_Q, 0], [\text{optimal}_Q, \text{Profit} (\text{optimal}_Q)], \text{Color} = \text{RGB}::\text{Green}) \);
• \( \text{plot}(f1,f2, \text{AxesTitles} = ["Q", "Profit"]); \)

![Figure 3. Locating the optimal level out output by plotting the profit function.](image)

In any case, the graphical methods only locate the profit maximizing level of output approximately, apparently a value slightly bigger than \( Q = 45 \). Using the MR=MC rule produced a more accurate result \( Q = 45.45454545 \). The profit maximizing level of output can also be located by using the profit function directly

• \( \text{delete optimal}_Q := \)

Take the first derivative of the profit function

• \( Df := D(\text{Profit}) ; \)
  \[ (Q \rightarrow 1000 - 2 \cdot Q) - (Q \rightarrow 20 \cdot Q) \]
Find the level of output that makes $D_f = 0$.

- $\text{ans:=solve}(D_f(Q)=0,Q) ;$
  \[
  \{ \frac{500}{11} \}
  \]

- $\text{optimal}_Q:=\text{float( op(ans) )};$
  \[
  45.45454545
  \]

The optimal level of output, $\text{optimal}_Q$, is a stationary point. It could be a maximum or a minimum. An indication of whether this is a maximum can be had by computing profit at the optimal level of output and at points near the optimal level. If the optimal level of output is a maximum, then the value of the profit function should be larger here than at nearby points.

- $\text{Profit}(\text{optimal}_Q) ;$
  \[
  22727.27273
  \]

- $\text{Profit}(\text{optimal}_Q-0.001) ;$
  \[
  22727.27272
  \]

- $\text{Profit}(\text{optimal}_Q+0.001) ;$
  \[
  22727.27272
  \]

The value of profit is larger at the optimal level of $Q$ than at the selected nearby points and this suggests that a maximum has been located. Taking the second derivative and evaluating it will determine whether $\text{optimal}_Q$ is a point that generates a maximum level of profit or a minimum level of profit.

- $D( D(\text{Profit})) ;$
  \[-22\]

Because the value of the second derivative is negative $\text{optimal}_Q$ is a level of output that generates a maximum level of profit for the monopolist.