MuPAD Primer

Luke Olsen and Max Jerrell

Introduction
This document is intended to provide the necessary hints and tools to allow undergraduate economics students with a familiarity with college algebra to use MuPAD to construct economic models and solve economic problems. If you want to take a closer look at any of these commands or to try to find a command that is not listed, use the MuPAD help browser (keyboard shortcut: F2).

MUPAD is a computer algebra system (CAS). It operates on symbols as well as number. X is certainly a symbol. Is 2 a symbol or a number? In MuPAD, it could be both.

This document is organized along the following topics:
• Examples of elementary math operations
• MuPAD specific techniques
• Assigning values to variables
• Defining functions in MuPAD
• The Solve Command
• The Assume Command
• Derivatives and Differentiation
• Plotting graphs with MuPAD
• Removing Braces (curly brackets) and The op Command

The sections should provide an adequate guide for creating and working with the economic models. Also available for reference are examples of complete projects. A few of the sections might not be necessary for the models assigned, but provide material for anyone interested in further exploring MuPAD's capabilities.
Examples of Elementary Math Operations in MuPAD
The following are examples of running elementary math operations. They are similar to what you would enter in a graphing calculator.

**Addition**
- \(2+2\)  
  \[= 4\]  
- \(x+x\)  
  \[= 2x\]  
- \(x+y\)  
  \[= x+y\]

**Subtraction**
- \(3-2\)  
  \[= 1\]  
- \(x-x\)  
  \[= 0\]  
- \(x-y\)  
  \[= x-y\]

**Multiplication**
- \(2*4\)  
  \[= 8\]  
- \(x*x\)  
  \[= 2x\]  
- \(x*y\)  
  \[= xy\]

**Division**
• 8/2
   4

• x/x
   1

• x/y
   x
   -
   y

Exponents
• x^2
   2
   x

• (x+1)^2
   2
   (x + 1)

• (x/y)^2
   2
   x
   -
   2
   y

• (x^2)^3
   6
   x

Square Root
• sqrt(9)
   3

Absolute Value
• abs(-99)
More Operations
This software is capable of more basic math operations, including trigonometric functions, \( \ln() \) and \( \log() \), inequalities \(<\), \( \geq \), \( >\), \( \leq \), as well as transcendental numbers like \( E \) (2.7182818...) and \( \pi \) (3.141592654...).

Derivatives and Differentiation
The derivative or differential of a function is the rate of change of that function. Graphically, the derivative is the slope of the function at the point at which you are measuring it.

An economic example that can be understood best as a derivative is the marginal propensity to consume (MPC). MPC is the rate at which consumption changes as income changes. Symbolically, \( \text{MPC} = \frac{\Delta C}{\Delta Y} \), where \( \Delta C \) is change in consumption and \( \Delta Y \) is change in income.

In MuPAD, there are two commands that you can use to find the derivative, which we will discuss in the next section.

MuPAD Specific Techniques
Here are a couple of techniques to improve the time using MuPAD.

Multiple commands
You can enter several commands on a line if they are separated by a semicolon.

- \( 2+2; \ 2/3 \ ; \ 3*(2/3); \)
  
  \[
  \begin{align*}
  4 \\
  2/3 \\
  2 
  \end{align*}
  \]

Suppress output
You can suppress output by using a colon instead of a semicolon.

- \( 2+2: \ 2/3 \ : \ 3*(2/3); \)

Fractions and Decimals
MuPAD treats 2 and 3 as symbols. To make MuPAD compute the result as a number use decimal notation. Compare:

- \( 2/3; \)
  
  \[
  \begin{align*}
  2/3 
  \end{align*}
  \]
Assigning Values to Variables
It is often useful to name a variable. Symbols can be assigned values using the `:=` operator.

- \( x := 2; y := 3; x + y; x / y; \)

\[
\begin{align*}
2 & \\
3 & \\
5 & \\
\frac{2}{3} & \\
\end{align*}
\]

As we mentioned above, if you want the result displayed in numerical form, use decimal notation:

- \( x := 2.0; y := 3.0; x + y; x / y; \)

\[
\begin{align*}
2.0 & \\
3.0 & \\
5.0 & \\
0.666666667 & \\
\end{align*}
\]

Defining Functions In MuPAD
Consider the function,

\[ f(x) = x^2. \]

If we want to know the value of \( f(x = 2) \), our function becomes

\[ f(2) = 2^2 = 4. \]

Functions have the same meaning in MuPAD. Here is how the above function can be created in MuPAD:
\[ f(x) = x^2 \]

then you can plug in values of \( x \) to find values of the function:

- \( f(2) \):
  \[ 4 \]

- \( f(3) \):
  \[ 9 \]

- \( f(y) \):
  \[ y \]

One of the chief reasons for using functions is to avoid typing the same information repeatedly. Consider:

\[ f(x) = \ln(x) \cdot e^x + 100 \cdot \sin(x) \]

- \( f(4.0) \):
  \[ 0.008857987733 \]

- \( f(10.0) \):
  \[ 50663.40968 \]

It is not necessary to type out the entire expression each time a value of \( f(x) \) is needed.

**The Solve Command**

We will often use MuPAD to solve systems of equations:

- \( \text{solve} \{(x+y=3, x-y=1), \{x, y\}\} \)

  \[ \{[x = 2, y = 1]\} \]
This situation specifies the `solve()` command to solve the system of equations:

\[
\begin{align*}
x + y &= 3 \\
x - y &= 1
\end{align*}
\]

for both \(x\) and \(y\) and it returned the solution \(x = 2, y = 1\). We could also have it solve for only one of those variables. MuPAD can also solve more complex systems of equations with no trouble. Consider a larger system of equations:

- \(eq1:=x+4*y+z+4*w=1;\)
- \(eq2:=-3*x+2*y+2*z+3*w=2;\)
- \(eq3:=3*x-2*y+3*z+2*w=3;\)
- \(eq4:=4*x+y-4*z-w=1;\)
- \(\text{solve}({eq1,eq2,eq3,eq4},{x,y,z,w});\)

\[
\begin{align*}
4w + x + 4y + z &= 1 \\
3w - 3x + 2y + 2z &= 2 \\
2w + 3x - 2y + 3z &= 3 \\
x - w + y - 4z &= 1
\end{align*}
\]

\{[w = 35/12, x = -5/12, y = -25/12, z = -23/12]\}

Solving this system by hand is not rocket science, but would be time consuming.

**The assume command**

Consider the following example:

- \(\text{solve}(x^2=1,x);\)

\{-1, 1\}

which give two results, namely that \(x\) could be minus or plus one. It might be the case that we only want to consider positive values so we could restrict the values \(x\) can take on by using the assume statement.

- \(\text{assume}(x>0);\)

\[> 0\]

- \(\text{solve}(x^2=1,x);\)

\{1\}
In economics we know that many things should only have positive values such as income, prices, nominal interest rates, consumption, hours worked, etc. If we provide MuPAD with this information it can greatly reduce the number of solutions that MuPAD produces for some problems. MuPAD is quite sophisticated and can produce seemingly weird results that are actually correct. Consider:

- \( \text{solve}(x^4=8, x); \)

\[
\frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \\
\{8 \quad , \quad -8 \quad , \quad -8i \quad , \quad 8i \}
\]

which gives four solutions, two of which are imaginary--indicated with the I in the solution--and two of which are real. If we want to restrict MuPAD to have only real solutions for \( x \) we can instruct it using the following assume command.

- \( \text{assume}(x, \text{Type::Real}); \)

\[
\text{Type::Real}
\]

- \( \text{solve}(x^4=8, x); \)

\[
\frac{1}{4} \quad \frac{1}{4} \\
\{8 \quad , \quad -8 \}
\]

and the imaginary solutions have gone away. We can further restrict \( x \) to only have positive values using

- \( \text{assume}(x>0, \_\text{and}); \)

\[
> 0
\]

where the \_and part combines Type::Real and \( x>0 \), otherwise Type::Real would be replaced with \( >0 \).

- \( \text{solve}(x^4=8, x); \)

\[
\frac{1}{4} \\
\{8 \}
\]

which gives the only positive, real answer. We will use assume frequently.

**Derivatives and Differentiation**

As we mention above in the Examples of elementary math operations section, the derivative of a function measures the rate of change of a function. When we take the derivative of the function at a particular point, we measure the rate of change of a function at that point. Again, this rate of change at a particular point is described graphically as the slope of the function at that point.

The functions in MuPAD are the \( \text{diff}(f, x) \) and \( \text{D}(f) \), where \( f \) is the function you want to differentiate and \( x \) is the variable by which you are differentiating. Here is an
example from MuPAD:

demand:=q->-2*q+6

\[ q \rightarrow 6 - 2q \]

\[ \text{diff(demand(q),q)} \]

\[-2\]

\[ \text{D(demand)} \]

\[-2\]

We defined our demand function to be \( P = -2q + 6 \). Notice that the slope of this function is \(-2\). When we use the \text{diff()}\ command, we specify the independent variable by which we differentiate. In this case, the independent variable is \( q \), quantity demanded. When we use the \text{D()}\ command, we do not specify the independent variable to differentiate.

When we come across a function with more than one independent variable, we must be careful. For more information on these commands, refer to the MuPAD help manual.

Plotting Graphs with MuPAD

Functions make it easy to plot items in MuPAD. To obtain a plot of a function, use \text{plotfunc2d(f,x=a..b)}\ command; where \( f \) is the name of the function to plot, \( x \) is the independent variable, \( a \) is the lower value of \( x \), and \( b \) is the upper value of \( x \). Consider the following example:

\[ \text{function1:=q->0.3*q+1; function2:=q->-0.3*q+36;} \]

\[ q \rightarrow 0.3q + 1 \]

\[ q \rightarrow 36 - 0.3q \]

Now, we ask MuPAD to graph function1, from \( q = [1, 10] \).

\[ \text{plotfunc2d(function1(q),q=0..10);} \]

MuPAD then produces this graph in another window:
We can also use MuPAD to graph multiple functions at once:

- `plotfunc2d(function1(q), function2(q), q=0..100);

Notice here that we extended the graph along the horizontal axis in order to visually spot the point at which the functions cross.
MuPAD can graph more exciting functions as well:

![Graph of 100*sin(x)+ln(x)*exp(x), 5*x^2*sin(5*x)](image)

**Figure 3**

Removing Braces (curly brackets) and The op Command

Consider solving the following equation:

- `assume(a>0);`  
  > 0

- `eqn1:=x/a=1;`  
  \[ \frac{x}{a} = 1 \]

- `x:=solve(eqn1,x);`  
  \[ \{a\} \]

- `x;`  
  \[ \{a\} \]

Note that the solution is included in a set of braces, or curly brackets `{ }`. Many times equations have multiple solutions as in the following example.

- `solve(z^2=a,z);`  
  \[ \left\{ a^{1/2}, -a^{1/2} \right\} \]

which has two solutions. What is included in the braces is a set of solutions. The problem with this is that we often can’t operate on a set of solutions in the
same fashion as we might operate on ordinary numbers.

For example:

- \( x; \) \{a\}
- \( \text{solve}(y=x,y); \)
  Error: Both sides of equation must be arithmetical expressions
  \[\text{[solve]}\]

The problem is that \( x \) is not an arithmetical expression but a set which could contain several expressions. We can extract a member out of the set using the \( \text{op} \) command, however, and use it as a regular expression

- \( \text{assume}(a>0); \)
  > 0
- \( \text{eqn1}:={x/a}=1; \)
  \( x \)
  \(- = 1 \)
  \( a \)
- \( x:=\text{solve}(\text{eqn1},x); \) \{a\}
- \( x; \) \{a\}
- \( x:=\text{op}(x); \)
  \( a \)

Note that the braces have disappeared. Using this member of the set we can now solve the expression
- \( \text{solve}(y=x,y); \) \{a\}

Note the solution is another set but that is what solve produces - sets of solutions.
The assign Command
This section shows how the MuPAD assign statement is used to select specific solutions out of a set of solutions.

First define some equations:
- eqn1:=x^2+y^2=a ;
- eqn2:=x^2-y^2=a;
- a:=3;

\[ \begin{align*}
2 \quad 2 \\
x \quad + \quad y \quad = \quad a \\
2 \quad 2 \\
x \quad - \quad y \quad = \quad a \\
3
\end{align*} \]

Now solve the system of equations and place the results in identifier solution:
- solution:=solve({eqn1,eqn2},{x,y});

\[ \left\{ \begin{array}{l}
1/2 \\
\left[ x = 3^{1/2}, \quad y = 0 \right], \left[ x = -3^{1/2}, \quad y = 0 \right] 
\end{array} \right. \]

The system of equations has two solutions. The following statement assigns the values of the first solution to identifiers x and y.
- assign(op(solution,1)):x,y;

\[ \begin{align*}
1/2 \\
3^{1/2}, \quad 0
\end{align*} \]

Display x and y to show that they do have the values of the first solution
- x;

\[ 3^{1/2} \]
- y;

The equations now have a numerical result, for example
- eqn1;

\[ 3 = 3 \]
This does provide a check on the solution, to wit $x^2+y^2$ is equal to 3, but we may want to restore the original situation.

- delete $x,y$;
- eqn1;

$$
\begin{align*}
2 & & 2 \\
& x^2 + y^2 = 3
\end{align*}
$$

Change the problem

- $a:=4$;

$$
4
$$

- eqn1;

$$
\begin{align*}
2 & & 2 \\
& x^2 + y^2 = 4
\end{align*}
$$

Solve the new problem but this time select the second solution:

- solution:=solve({eqn1,eqn2},{x,y});
- assign(op(solution,2)):x,y;

$$
\begin{align*}
\{ [x = -2, y = 0], [x = 2, y = 0] \}
\end{align*}
$$

Display $x$ and $y$ to see if it worked

- $x;y$;

$$
\begin{align*}
2 & & 0
\end{align*}
$$